Optimal quota for sector-specific immigration

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Abstract

Sectoral labor supply shortage is a cause of concern in many OECD countries and has raised support for immigration as a potential remedy. In this paper, we derive a general equilibrium model with overlapping generations, where natives require a compensating wage differential for working in one sector rather than in another. We identify price and wage effects of immigration on three different groups of natives: the young working in one of two sectors and the old. We determine the outcome of a majority vote on immigration into a given sector as well as the social optimum. The main findings are that i) the old determine any majority voting outcome of non-zero immigration into both sectors, ii) socially optimal immigration is smaller than or equal to the majority voting outcome, and iii) immigration is not necessarily a substitute for native mobility across sectors.

Key words: immigration, political economy, welfare, sectoral mobility.

JEL codes: F22, J31, J61.

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1 Introduction

Immigration policy is one of the most pressing issues in developed countries that face large and growing numbers of migrants, and it is just as controversial. While immigration is opposed because of expected negative wage effects on native workers, it is also often promoted as a way to alleviate labor shortages in specific labor market segments, skill- or occupation-wise. In a situation of excess labor demand, an increase of labor supply via immigration is an alternative to an increase in prices to achieve labor market equilibrium. The existing literature on the labor market effects of immigration typically studies wage effects in a one-sector economy, but does not consider price effects of immigration (Card 2001, Borjas 2003, Ottaviano and Peri 2006). However, recent empirical contributions by Cortes (2006) for the U.S. or Blanchflower and Shadforth (2009) for the UK show that immigration can have important attenuating effects on prices. Felbermayr and Kohler (2007) derive wage and price effects of immigration in a theoretical framework, taking immigration policy as given.

Optimal immigration policy that emerges as a result of different effects of immigration on natives is the subject of a number of important contributions to the economic immigration literature. For example, Benhabib (1996) and Facchini and Willmann (2005) derive policies that select immigrants by the complementarity of their factors (capital, skilled or unskilled labor) in a majority vote and a game among agencies, respectively, in order to maximize natives’ income. Storesletten (2000) determines the age and skill requirements for immigrants that maximize the gain for the public budget.

The contribution of this paper is to derive optimal policies that select immigrants by sector, taking into account both wage and price effects of immigration. Any immigration policy that aims at increasing native welfare has to consider both effects, as it is real wages, not nominal ones, which determine the overall welfare impact of immigration on natives. Existing immigration policies reflect these concerns. A number of countries identify occupational shortages regularly and use these as criteria to favor or facilitate immigration. In Europe, a standard procedure is to subject potential immigrants to an employment test, where an employer needs to declare his need of the immigrant for a job that cannot be filled by any resident qualified candidate. In the United Kingdom, a sectors-based migrant worker scheme was introduced in 2002 for low-skill jobs in the sectors of hotels and catering, and food manufacturing. In Australia, potential immigrants in required occupations receive extra points in the immigrant selection process.

To model observed occupational or sector-specific labor shortage as described above, we analyze a labor market that is segmented into two sectors, where natives exhibit sector-specific work preferences: for given job characteristics, they require a compensating wage differential (compare Rosen 1986) for working in

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1See OECD (2006).
2In nursing occupations, for example, labor market shortages are already present in many countries and are likely to expand with the ageing of native populations.
4Miller (1999).
one sector rather than in the other.\textsuperscript{5,6} In reality, job preferences are only one reason for wage differentials existing in the absence of productivity differences. Our model applies equally to labor supply shortage due to geographic moving costs between sectors, costs associated with the loss of sector-specific human capital or the necessary acquisition of new human capital.\textsuperscript{2} Natives are heterogeneous in terms of the amount of their required wage differential or moving cost.\textsuperscript{8} This is the main difference in our model between natives and migrants. Since we are interested in selective immigration policies targeted at specific sectors or occupations, we assume that natives and migrants are homogeneous within sectors.

The second key feature of our model is an overlapping generations structure. We derive optimal sector-specific immigration for three different groups of natives, the young working in one of two sectors, and the old. Intuitively, one would expect that old (retired) generations have a stronger preference for migration than young generations, since they do not experience potentially negative wage effects, but only positive price effects. Further, one would expect that old generations do not have a strong preference for migration to be specific to one or the other sector of the economy, while young generations should have such preferences, since their wages are directly affected by the sector-specific structure of the migration flow. We show that the old support immigration into both sectors, while the young oppose immigration into ‘their’ sector. However, the attitude of the young will be different, if they can move from one sector to the other as a response to a change in wage differentials due to immigration, given their sector-specific work preference. We derive the optimal amount of sector-specific immigration under two regimes, one where the sector choice of workers is fixed, and one where it is endogenous to migration.

Given that different groups are affected differently, what is the likely outcome on sector-specific immigration policies? We employ majority voting as a simple means of aggregating individual preferences and find that, in the case of fixed sector choice, immigration quota are strictly positive in both sectors. In the case of endogenous sector choice immigration quota can be zero, if the young in both sectors form a majority against immigration into both sectors. This is possible because they now experience a negative wage effect not only from immigration into their own sector but also from immigration into the other sector. If this negative wage effect is small enough, we find that immigration quota are strictly positive and the same as in the case of fixed sector choice. We perform numerical simulations to derive immigration quota for a range of existing parameter value estimates and compare the politically determined quota to the quota that maximize social welfare. We find that socially optimal immigration quota are smaller than those determined in a majority voting outcome, if the voting outcome is determined by the preferences of the old, who do not take the negative effect of immigration on wages into account. Socially optimal immigration is zero, if there is a majority against immigration.

\textsuperscript{5}Klaver and Visser (1999) find for different sectors in the Dutch economy that their image is not good enough, at the going wage rate, to attract a sufficient number of workers, even if supply is abundant. Compare OECD (2003, p. 104).

\textsuperscript{6}Borjas (2007) mentions in his blog the health sector in the UK as an example of a ‘low-wage ghetto’, which ‘drives natives into alternative, better-paid options and fulfills the prophecy that there are some jobs that natives just won’t do.’

\textsuperscript{7}According to Zimmermann et al. (2007), there is evidence for regional and sectoral wage differentials within occupations, which suggests that mobility is insufficient even within a relatively homogeneous labor market (see DeNew and Schmidt 1994, Möller and Bellmann 1996 and Haisken-DeNew and Schmidt 1999 for Germany).

\textsuperscript{8}We could also think of the two sectors as a low- and a high-skill sector, with natives being heterogeneous with regard to their cost of investing in human capital.
Finally, we compare the welfare effects of immigration in the two regimes of fixed and endogenous sector choice of natives. In particular, one might expect that sector-specific immigration is a substitute, in terms of social welfare, for native mobility across sectors. This expectation is inherent in a number of recent policy proposals that suggest enhancing the mobility of native workers as an alternative to immigration when dealing with labor shortages. However, depending on relative labor supply in the two sectors, it is possible that the welfare effect of immigration is higher with than without native mobility. We identify conditions under which the substitutability result does, and does not, hold.

2 The model

Consider a two-country, two-period overlapping-generations economy. Every agent is born with one unit of inelastically supplied labor. Labor is perfectly substitutable within sectors. The labor endowments of countries I and II are \( N > 0 \) and \( M > 0 \), respectively, and constant over time.\(^9\)

2.1 Production

In country I, there are two production sectors. The output of sector A \((X_A)\) is non-tradable and hence can be consumed only in country I. The output of sector B \((X_B)\) is tradable and hence can be consumed in both countries. In country II, there is only one sector whose output \((X_C)\) is tradable. The production in each sector is subject to constant-returns-to-scale technology:

\[
X_i = L_i^{\gamma_i}K_i^{1-\gamma_i}, \quad \gamma_i \in (0,1), \quad i \in \{A,B,C\}
\]  

where \( L_i \) and \( K_i \) denote labor and capital input used by sector \( i \), respectively. Each sector is under perfect competition: the unit price of a production factor is equal to the value of its marginal product. Therefore, the sectoral wages are

\[
w_i \equiv p_i \frac{\partial X_i}{\partial L_i} = p_i \gamma_i \left( \frac{K_i}{L_i} \right)^{1-\gamma_i}
\]  

where \( p_i \) is the unit price of the output by sector \( i \). We assume perfectly mobile capital such that there is only one interest rate that is equal to the value of the marginal product of capital in every sector, i.e., \( \forall i \),

\[
r \equiv p_i \frac{\partial X_i}{\partial K_i} = p_i (1 - \gamma_i) \left( \frac{L_i}{K_i} \right)^{\gamma_i}
\]

\(^9\)Compare Zimmermann et al. (2007), p.73. They suggest financial incentives such as tax exemptions for relocation or commuting costs as a way to increase inter-sectoral native mobility. The same idea was pronounced by the Austrian minister for economics and labor in 2007.

\(^{10}\)The set-up of a dynamic model that would allow to consider population growth over time could be an interesting avenue for further research.
2.2 Consumption

Each agent in country I has the following inter-temporal utility function:

\[ u_I(x(1)) + \frac{1}{1+\delta} u_I(x(2)) \]  

(4)

where \( \delta \geq 0 \) is the common rate of discounting future consumption; \( x(1) = (x_A(1), x_B(1), x_C(1)) \) is the first-period consumption bundle; and \( x(2) = (x_A(2), x_B(2), x_C(2)) \) is the second-period consumption bundle. It implies that utility is invariant and additive over time. We assume \( \partial u_I/\partial x_i > 0 \) and \( \partial^2 u_I/\partial^2 x_i < 0 \). More specifically,

\[ u_I(x) \equiv \alpha \ln x_A + \beta \ln x_B + (1 - \alpha - \beta) \ln x_C, \quad \alpha, \beta, \alpha + \beta \in (0, 1). \]  

(5)

The objective of the agent is to maximize the utility function subject to \( p(1) \cdot x(1) \leq w - s \) and \( p(2) \cdot x(2) \leq (1 + r) s \) where \( s \) is savings. Due to non-satiating utility and no bequest, the combined budget constraint is \( p(1) \cdot x(1) + p(2) \cdot x(2) / (1 + r) = w \).

We assume that the interest rate to the first-period savings is paid in the very beginning of the second period. The first-period savings are used as capital input in the first period, and the interest rate is determined via equation (3) in the same period.

For each agent in country II, we replace \( u_I \) in (4) by

\[ u_{II}(x) \equiv \theta \ln x_B + (1 - \theta) \ln x_C, \quad \theta \in (0, 1) \]  

(6)

because the output of sector \( A \) in country I is not tradable.

Accordingly, we obtain the following demand functions:

\[ x_A^A(1) = e \frac{w_A(1)}{p_A(1)}, \quad x_A^A(2) = (1 - e) \frac{w_A(1)}{p_A(2)} (1 + r(1)), \]
\[ x_B^A(1) = e \frac{w_B(1)}{p_B(1)}, \quad x_B^A(2) = (1 - e) \frac{w_B(1)}{p_B(2)} (1 + r(1)), \]
\[ x_C^A(1) = e (1 - \alpha - \beta) \frac{w_A(1)}{p_C(1)}, \quad x_C^A(2) = (1 - e) (1 - \alpha - \beta) \frac{w_A(1)}{p_C(2)} (1 + r(1)), \]
\[ x_A^B(1) = e \frac{w_A(1)}{p_A(1)}, \quad x_A^B(2) = (1 - e) \frac{w_A(1)}{p_A(2)} (1 + r(1)), \]
\[ x_B^B(1) = e \frac{w_B(1)}{p_B(1)}, \quad x_B^B(2) = (1 - e) \frac{w_B(1)}{p_B(2)} (1 + r(1)), \]
\[ x_C^B(1) = e (1 - \alpha - \beta) \frac{w_A(1)}{p_C(1)}, \quad x_C^B(2) = (1 - e) (1 - \alpha - \beta) \frac{w_A(1)}{p_C(2)} (1 + r(1)), \]
\[ x_A^C(1) = 0, \quad x_A^C(2) = 0, \]
\[ x_B^C(1) = e \frac{w_C(1)}{p_B(1)}, \quad x_B^C(2) = (1 - e) \frac{w_C(1)}{p_B(2)} (1 + r(1)), \]
\[ x_C^C(1) = e (1 - \theta) \frac{w_C(1)}{p_C(1)}, \quad x_C^C(2) = (1 - e) (1 - \theta) \frac{w_C(1)}{p_C(2)} (1 + r(1)), \]

where \( e \equiv (1 + \delta) / (2 + \delta) \) and the superscript indicates the employment sector, e.g., \( x_A^B(1) \) is the first-period demand for the output of sector \( A \) by an agent who is employed in sector \( B \). In the second period,
agents use their first-period savings plus interest for consumption.

2.3 Factor supply

The demand functions imply that individual savings \( s_i \) form a fixed fraction \( 1 - e \) of each agent’s labor income \( w_i \). Each agent inelastically supplies one unit of labor in the first period of life. Accordingly, in each period, total capital is equal to total savings given by

\[
\sum_i K_i = \sum_i s_i L_i = (1 - e) \sum_i w_i L_i. \tag{7}
\]

In country I, agents choose one of the two national sectors for work in the first life-period. Let \( \omega_h \in (-\infty, \infty) \) denote the wage differential between sectors A and B required by young agent \( h \) to work in sector A. Young agent \( h \) chooses to work in sector A, if \( w_A - \omega_h > w_B \) and in sector B otherwise. We assume a continuous cumulative distribution function \( \Phi(\cdot) \) of young agents with respect to the required wage differential.\(^{11}\) Denote the wage differential by

\[
\omega \equiv w_A - w_B. \tag{8}
\]

Since young agent \( h \) with \( \omega_h < \omega \) chooses to work in sector A, \( \Phi(\omega) \) gives the fraction of country I’s young native population choosing to work in sector A.

We assume that the wage in country II is low compared to country I such that country I can face a large number of country-II workers who want to migrate to either sector A or B. Country I can decide on the number of immigrant workers in sectors A and B. By doing so, the country decides on the number of workers in sector \( C \) residually, since we assume an inelastic supply of labor and full employment.

Accordingly, at given wage rates and a given number of migrants, labor supply in each sector is given by

\[
egin{align*}
L_A &= N \Phi(\omega) + M_A \geq 0 \tag{9} \\
L_B &= N (1 - \Phi(\omega)) + M_B \geq 0 \tag{10} \\
L_C &= M - M_A - M_B \geq 0 \tag{11}
\end{align*}
\]

Let \( N\Phi(\omega) \equiv N_A \) and \( N(1 - \Phi(\omega)) \equiv N_B \) in the following.

2.4 Equilibrium

We set the supply of each sector’s output given by equation (1) equal to each sector’s demand expressed by the individual demand functions times the respective population sizes given by equations (9)-(11). In

\(^{11}\)Let us assume the corresponding density function is non-degenerate, i.e. \( \phi(\cdot) > 0 \forall \omega_h \).
any given period \( t \), the following relationships hold:

\[
X_A(t) = e^{\alpha w_A(t)} L_A(t) + w_B(t) L_B(t) \frac{p_A(t)}{p_A(t)} + (1 - e) \alpha w_A(t-1) L_A(t-1) + w_B(t-1) L_B(t-1) (1 + r(t-1))
\]

\[
X_B(t) = e^{\beta w_A(t)} L_A(t) + w_B(t) L_B(t) \frac{p_B(t)}{p_B(t)} + (1 - e) \beta w_A(t-1) L_A(t-1) + w_B(t-1) L_B(t-1) (1 + r(t-1))
\]

\[
X_C(t) = e^{(1 - \alpha - \beta) w_A(t)} L_A(t) + w_B(t) L_B(t) \frac{p_C(t)}{p_C(t)} + e(1 - \theta) w_C(t) L_C(t) \frac{p_C(t)}{p_C(t)} + (1 - e) \gamma w_A(t-1) L_A(t-1) + w_B(t-1) L_B(t-1) (1 + r(t-1))
\]

By substituting wages given by (2) into these and solving the system, we obtain the equilibrium prices:

\[
p_i(t) = \frac{\psi_i(t)}{X_i(t)}
\]

where \( \psi_i(t) \) depends only on \( r \) and \( \psi_i \) in the previous period \( t - 1 \).\(^{12}\) This in turn implies that the value of output in each sector \( p_i(t) X_i(t) \) is predetermined in any period: a change in labor supply in a given sector due to immigration changes the output and the price in that sector such that the value of output remains constant \( \psi_i(t) \).\(^{13,14}\)

Lemma 1 Immigration affects neither the total amount of capital in the world, nor its distribution across sectors, nor the interest rate.

Proof. By substituting (15) into (3), we get

\[
\frac{K_i}{K_j} = \frac{\psi_i 1 - \gamma_i}{\psi_j 1 - \gamma_j}, \quad i, j \in \{A, B, C\}, \quad i \neq j
\]

\(^{12}\)The expression for \( \psi_i(t) \) is too long for inclusion in the main text and is left to the Appendix.

\(^{13}\)This result remains robust in case of different degrees of total factor productivity across sectors.

\(^{14}\)The value of output changes, if the elasticity of substitution between factors is unequal to one. We stick to the simpler functional form as our benchmark, as expressions become too complex for an analytical derivation of results otherwise.
for any given period. The capital ratio between sectors is thus predetermined. The resource constraint (7) implies, by using (2) and (15),

\[
\sum_i K_i = (1 - e) \sum_i \gamma_i \psi_i
\]

for any given period, which shows that total capital in each period is a constant share of the sectoral output values \(\psi_i\) in that period, which, in turn, depend on the values of output in the previous period. This implies that total capital and labor supply in the initial period completely determine total capital in subsequent periods.

Since total capital only depends on the value of sectoral output, which is predetermined, the interest rate is fixed. An increase in \(L_A\) due to an increase in immigration would be completely offset by a decrease in \(p_A\) and, accordingly, in wages and savings. Immigration does not change total capital and, therefore, the interest rate, because while it increases the number of savers in country I, savings per capita fall due to the effect of a price decrease on the wage. Immigration increases the physical marginal product of capital, but not its value. Note that this is in contrast to models that abstract from sector- or country-specific prices such as Ottaviano and Peri (2006), where it is total capital and not prices that adjusts to immigration.

3 Majority voting outcomes

In the following, we examine the outcome of a referendum on immigration into sectors A and B in country I. Thereby, we distinguish between two cases: one where native workers are immobile across sectors and one where they are mobile across sectors. As immigration changes the wage differential between sectors A and B, it will also change the distribution of natives across sectors, if it is costless to switch (workers are mobile). Natives will not switch sector, if the cost is too high (workers are immobile).\(^{15}\)

3.1 Immobile workers

**Proposition 1** Assume young natives are immobile across sectors. Then, in the absence of an absolute majority of any one group, majority voting on immigration in the two sectors results in immigration quota that are preferred by the old and strictly positive.

\(^{15}\)Compare Dixit and Rob (1994) for a model of endogenous labor immobility across sectors due to switching costs. In the model, incentives to switch do not arise from immigration but from technological shocks that affect wages in the two sectors differently.
Proof. We derive individual indirect utility of young natives in country I by substituting the demand functions, wages (2) and prices (15) into the utility function (4). We have, for $i \neq C$ and $t = 1$,

$$v^y_i(1) \equiv \alpha \ln \left( e^{\alpha \frac{w_i(1)}{p_A(1)}} \right) + \beta \ln \left( e^{\beta \frac{w_i(1)}{p_B(1)}} \right) + (1 - \alpha - \beta) \ln \left( e^{(1 - \alpha - \beta) \frac{w_i(1)}{p_C(1)}} \right)$$

$$+ \frac{1}{1+\delta} \left[ \alpha \ln \left( (1 - e) \frac{w_i(1)}{p_A(2)} (1 + r(1)) \right) + \beta \ln \left( (1 - e) \frac{w_i(1)}{p_B(2)} (1 + r(1)) \right) + (1 - \alpha - \beta) \ln \left( (1 - e) (1 - \alpha - \beta) \frac{w_i(1)}{p_C(2)} (1 + r(1)) \right) \right]$$

where superscript $y$ denotes young.

The indirect utility of a retired agent is simply the one-period lag of the fourth term in the above expression without the discounting:

$$v^o(1) \equiv \alpha \ln \left( (1 - e) \frac{w(0)}{p_A(1)} (1 + r(0)) \right) + \beta \ln \left( (1 - e) \frac{w(0)}{p_B(1)} (1 + r(0)) \right) + (1 - \alpha - \beta) \ln \left( (1 - e) (1 - \alpha - \beta) \frac{w(0)}{p_C(1)} (1 + r(0)) \right),$$

where superscript $o$ denotes old.\(^{16}\)

For the first derivative of indirect utility of a young native in sector A with respect to immigration into the sector, we get

$$\frac{\partial v^y_A}{\partial M_A(1)} = \frac{1}{w_A(1)} \frac{dw_A(1)}{dM_A(1)} - \frac{\alpha}{p_A(1)} \frac{dp_A(1)}{dM_A(1)} - \frac{1 - \alpha - \beta}{p_C(1)} \frac{dp_C(1)}{dM_A(1)} + \frac{1}{(1 + \delta)w_A(1)} \frac{dw_A(1)}{dM_A(1)}$$

where the first and fourth terms are the wage effects. Note that the wage effect in the fourth term is felt in the second period of lifetime. The second term is the sector-A price effect, while the third term is the sector-C price effect. We can drop time indices and reduce the expression to

$$\frac{\partial v^y_A}{\partial M_A} = -\frac{1}{eL_A} + \frac{\alpha \gamma_A}{L_A} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}. \quad (20)$$

The first derivatives of the indirect utility of young natives in sector B and the old with respect to $M_A(1)$ are the same:

$$\frac{\partial v^y_B}{\partial M_A} = \frac{\partial v^o}{\partial M_A} = \frac{\alpha \gamma_A}{L_A} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}. \quad (21)$$

That is, immigration into sector A affects workers in sector B and the old only via the price effects: a decrease in the sector-A price and an increase in the sector-C price. Therefore, the wage effect represented by the first term of (20) is missing in (21).

\(^{16}\)We omit the sector subscript from $v$ and $w$ in (19) because only the three output prices are affected by immigration in this expression, which enter utility in the same way regardless of whether an agent worked in sector A or B in the previous period.
Analogously, regarding immigration into sector B, we have
\[
\frac{\partial v^B}{\partial M_B} = \frac{1}{e L_B} + \frac{\beta \gamma_B}{L_B} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C} \tag{22}
\]
and
\[
\frac{\partial v^A}{\partial M_B} = \frac{\partial v^0}{\partial M_B} = \beta \gamma_B \frac{L_B}{L_C} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}. \tag{23}
\]

Immigration into sector A (B) affects utility of workers already in the sector in three ways, namely via (i) a decrease in the wage (first term on the right-hand side of (20) and (22)), (ii) a decrease in the sector-A (B) price (second term) and (iii) an increase in the sector-C price (third term). The total effect is negative because the negative wage effect dominates the positive sector-A (B) price effect, i.e., \(1/e > \alpha \gamma_A\) and \(1/e > \beta \gamma_B\). Young natives in sector A (B) therefore oppose immigration into their own sector. Their optimal amount of \(M_A\) (\(M_B\)) is zero.

According to the first-order conditions (20)-(23), we know that - for given \(M_A\) - the young in sector A and the old prefer the same \(M_B\), which is greater than the \(M_B\) preferred by the young in sector B. Likewise - for given \(M_B\) - the young in sector B and the old prefer the same \(M_A\), which is greater than the \(M_A\) preferred by the young in sector A. Hence, the young in each sector form a majority with the old on immigration into the sector other than their own. As a result, the median voter represents the preference of the old in any one vote on \(M_A\) or \(M_B\).

Now, in a majority vote on a pair of immigration quota into the two sectors, the policy space is two-dimensional as voters vote simultaneously on both \(M_A\) and \(M_B\). Therefore, for the old to dominate the voting outcome, we need to show that there is no pair of immigration quota in the two sectors that will make a majority better off than the pair preferred by the old. For this, we compute the optimal pair \((M_A, M_B)\) for each group of voters as follows.

Young natives in sector A choose \(M_B\) by setting \(M_A = 0\) and solving
\[
\beta \gamma_B \frac{L_B}{L_C} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}. \tag{24}
\]

Assume that the population in country II is large relative to the population in country I. Then, the positive price effect initially dominates the negative price effect. Young natives support immigration into the sector that is not their own, and the old support immigration into both sectors. With an increase in \(M_A\) or \(M_B\), this positive net effect of immigration decreases and eventually becomes negative.
Young natives in sector B choose $M_A$ by setting $M_B = 0$ and solving
\[
\frac{\alpha \gamma_A}{L_A} = \frac{(1 - \alpha - \beta) \gamma_C}{L_C}.
\] (25)

Old natives choose $M_A$ and $M_B$ such that
\[
\frac{\alpha \gamma_A}{L_A} = \frac{\beta \gamma_B}{L_B} = \frac{(1 - \alpha - \beta) \gamma_C}{L_C}.
\] (26)

By substituting (9)-(11) into these, we find the choice by young natives in sector A is
\[
(M_A^y, M_B^y) = \left(0, \frac{\beta \gamma_B M - (1 - \alpha - \beta) \gamma_C N_B}{\beta \gamma_B + (1 - \alpha - \beta) \gamma_C}\right).
\] (27)

The choice by young natives in sector B is
\[
(M_A^y, M_B^y) = \left(\frac{\alpha \gamma_A M - (1 - \alpha - \beta) \gamma_C N_A}{\alpha \gamma_A + (1 - \alpha - \beta) \gamma_C}, 0\right).
\] (28)

The choice by old natives is a pair of
\[
M_A^o = \frac{\alpha \gamma_A [M + N_B] - [\beta \gamma_B + (1 - \alpha - \beta) \gamma_C] N_A}{\alpha \gamma_A + \beta \gamma_B + (1 - \alpha - \beta) \gamma_C},
\] (29)

and
\[
M_B^o = \frac{\beta \gamma_B [M + N_A] - [\alpha \gamma_A + (1 - \alpha - \beta) \gamma_C] N_B}{\alpha \gamma_A + \beta \gamma_B + (1 - \alpha - \beta) \gamma_C}.
\] (30)

Given $M_A^o \geq 0$ and $M_B^o \geq 0$, we find that $M_B^B > M_A^o \geq M_A^A$ and $M_B^A > M_B^o \geq M_B^B$. It follows that there is no pair of immigration quota that is preferred by a majority over the pair preferred by the old ($M_A^o$, $M_B^o$). In other words, there will always be a majority against any move from ($M_A^o$, $M_B^o$): the young in B (together with the old) are against a further increase in $M_B$ or a further decrease in $M_A$ or any positive linear combination of these and, likewise, the young in A (together with the old) are against a further increase in $M_A$ or a further decrease in $M_B$ or any positive linear combination of these. In this two-dimensional vote, a median voter exists because the tastes of voters who are not the median voter are diametrically opposed.\(^\text{18}\)

\[\text{■}\]

To sum up, in our model the young in both sectors oppose immigration into their respective sectors because such immigration depresses their wages: these negative effects dominate any positive price effect. However, they desire immigration into the other sector up to the point where the marginal benefit from immigration...
the decrease in that sector’s output price is equal to the marginal cost from the increase in the price of output produced in country II.

The old desire immigration into both sectors such that the marginal benefit from the decrease in the price of both sectors’ output is equal to the marginal cost from the increase in the price of output produced in country II. They do not care about the wage effect of immigration.\(^{19}\) The pair of immigration levels chosen by the old represents the median position because the young each form a majority with the old against any move from the pair of immigration levels preferred by the old. The majority voting outcome on immigration into sectors A and B is equal to \(M_A^o\) and \(M_B^o\) for \(M_A^o \geq 0\) and \(M_B^o \geq 0\).

3.2 Mobile workers

The effects of immigration on each of the three groups will be different, if workers are mobile and can switch sector in response to immigration. We compute the change in total sectoral labor supply using the expressions for labor demand (2) and labor supply (10)-(11). These determine the equilibrium distribution of migrant and native workers across sectors A and B. Labor supply in sectors A and B is defined implicitly by the following two equations:

\[
F_A \equiv N\Phi \left( \frac{\psi_A \gamma_A}{L_A} - \frac{\psi_B \gamma_B}{L_B} \right) + M_A - L_A = 0 \tag{31}
\]

\[
F_B \equiv N \left( 1 - \Phi \left( \frac{\psi_A \gamma_A}{L_A} - \frac{\psi_B \gamma_B}{L_B} \right) \right) + M_B - L_B = 0 \tag{32}
\]

where we substituted for \(\omega\) by using (8), (2) and (15).

**Lemma 2** Immigration into a given sector increases labor supply in both sectors A and B. But the increase in that given sector is larger when immigration enters into that sector than when it enters into the other sector.

**Proof.** Since

\[
\text{det} \left( \frac{\partial F_A}{\partial L} \frac{\partial F_A}{\partial L} \frac{\partial F_A}{\partial L} \frac{\partial F_A}{\partial L} \right) = 1 + N\Phi(\omega) \left( \frac{\psi_A \gamma_A}{L_A^2} + \frac{\psi_B \gamma_B}{L_B^2} \right) \neq 0,
\]

\(^{19}\)We abstract from public finance effects of immigration, which could represent either net benefits or net costs to both the native young as well as the old.
We apply Cramer’s rule to the system (31)-(32) to get

\[
\frac{\partial L_A}{\partial M_A} = \frac{1 + N\Phi(\omega)\frac{\psi_B\gamma_B}{L_B}}{1 + N\Phi(\omega)\left(\frac{\psi_A\gamma_A}{L_A} + \frac{\psi_B\gamma_B}{L_B}\right)} \in (0,1) \tag{33}
\]

\[
\frac{\partial L_B}{\partial M_A} = \frac{N\Phi(\omega)\frac{\psi_B\gamma_B}{L_B}}{1 + N\Phi(\omega)\left(\frac{\psi_A\gamma_A}{L_A} + \frac{\psi_B\gamma_B}{L_B}\right)} \in (0,1) \tag{34}
\]

\[
\frac{\partial L_A}{\partial M_B} = \frac{N\Phi(\omega)\frac{\psi_A\gamma_A}{L_A}}{1 + N\Phi(\omega)\left(\frac{\psi_A\gamma_A}{L_A} + \frac{\psi_B\gamma_B}{L_B}\right)} \in (0,1) \tag{35}
\]

\[
\frac{\partial L_B}{\partial M_B} = \frac{1 + N\Phi(\omega)\frac{\psi_A\gamma_A}{L_A}}{1 + N\Phi(\omega)\left(\frac{\psi_A\gamma_A}{L_A} + \frac{\psi_B\gamma_B}{L_B}\right)} \in (0,1) \tag{36}
\]

These expressions imply that \( \frac{\partial L_A}{\partial M_A} > \frac{\partial L_A}{\partial M_B} \) and \( \frac{\partial L_B}{\partial M_B} > \frac{\partial L_B}{\partial M_A} \).

As a response to immigration into a given sector, some natives choose to switch. Immigration into sector A, for example, decreases the wage in that sector such that the wage differential becomes too small for some to compensate them for working in sector A. They choose sector B rather than sector A for employment. Immigration into sector B, on the other hand, increases the wage differential, such that more natives are now willing to work in sector A than before. Immigration into a given sector thus causes a movement of natives away from that sector, partially offsetting the initial wage decrease. Immigration is not totally offset by the sector change of natives. This is because, while immigration into a given sector decreases the wage in that sector, the ensuing sector switch by natives results in a wage decrease also in the other sector. As a result, labor supply increases in both sectors, and wages and prices decrease in both sectors.\(^{20}\)

**Proposition 2.** Assume young natives are mobile across sectors. Then, in the absence of an absolute majority of any one group and in the absence of a majority against immigration or a voting cycle, majority voting on immigration in the two sectors results in immigration quota that are preferred by the old and the same as in case of sectoral immobility.

**Proof.** Expressions (18) and (19) imply the following derivatives with respect to immigration into sector A:

\[
\frac{\partial e^y_A}{\partial M_A} = \left(\alpha \gamma_A - \frac{1}{c}\right) \frac{1}{L_A} \frac{\partial L_A}{\partial M_A} + \beta \gamma_B \frac{\partial L_B}{\partial M_B} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C} \tag{37}
\]

\(^{20}\)Note that the distribution of natives across sectors resulting from immigration into sector A is different from the one resulting from immigration into sector B. This is because the change in the wage differential between sectors A and B is different (it decreases with \(M_A\) and increases with \(M_B\)).
\[
\frac{\partial v_B^o}{\partial M_A} = \frac{\alpha \gamma_A}{L_A} \left[ 1 + \frac{N_A \psi_B \gamma_B}{L_B} \right] + \left( \beta \gamma_B - \frac{1}{e} \right) \frac{1}{L_B} \left[ \frac{1}{L_B} \frac{\partial L_B}{\partial M_A} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C} \right]
\]

(38)

or, substituting using (33)-(36):

\[
\frac{\partial v_B^o}{\partial M_A} = \frac{\alpha \gamma_A}{L_A} \left[ 1 + \frac{N_A \psi_B \gamma_B}{L_B} \right] + \left( \beta \gamma_B - \frac{1}{e} \right) \frac{1}{L_B} \left[ \frac{N_A \psi_B \gamma_B L_B}{1 + N_A \left( \psi_A \gamma_A L_A + \psi_B \gamma_B L_B \right)} \right] - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}
\]

(40)

\[
\frac{\partial v_B^o}{\partial M_A} = \frac{\alpha \gamma_A}{L_A} \left[ 1 + \frac{N_A \psi_B \gamma_B}{L_B} \right] + \left( \beta \gamma_B - \frac{1}{e} \right) \frac{1}{L_B} \left[ \frac{N_A \psi_B \gamma_B}{1 + N_A \left( \psi_A \gamma_A L_A + \psi_B \gamma_B L_B \right)} \right] - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}
\]

(41)

\[
\frac{\partial v_B^o}{\partial M_A} = \frac{\alpha \gamma_A}{L_A} \left[ 1 + \frac{N_A \psi_B \gamma_B}{L_B} \right] + \beta \gamma_B \left[ \frac{N_A \psi_B \gamma_B L_B}{1 + N_A \left( \psi_A \gamma_A L_A + \psi_B \gamma_B L_B \right)} \right] - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}
\]

(42)

Analogously, with respect to immigration into sector B, we have

\[
\frac{\partial v_B^o}{\partial M_B} = \frac{\alpha \gamma_A}{L_A} \left[ 1 + \frac{N_A \psi_B \gamma_B}{L_B} \right] + \left( \beta \gamma_B - \frac{1}{e} \right) \frac{1}{L_B} \left[ \frac{1}{L_B} \frac{\partial L_B}{\partial M_B} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C} \right]
\]

(43)

\[
\frac{\partial v_B^o}{\partial M_B} = \frac{\alpha \gamma_A}{L_A} \left[ 1 + \frac{N_A \psi_B \gamma_B}{L_B} \right] + \left( \beta \gamma_B - \frac{1}{e} \right) \frac{1}{L_B} \left[ \frac{1}{L_B} \frac{N_A \psi_B \gamma_B}{1 + N_A \left( \psi_A \gamma_A L_A + \psi_B \gamma_B L_B \right)} \right] - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}
\]

(44)

\[
\frac{\partial v_B^o}{\partial M_B} = \frac{\alpha \gamma_A}{L_A} \left[ 1 + \frac{N_A \psi_B \gamma_B}{L_B} \right] + \beta \gamma_B \left[ \frac{1}{L_B} \frac{N_A \psi_B \gamma_B L_B}{1 + N_A \left( \psi_A \gamma_A L_A + \psi_B \gamma_B L_B \right)} \right] - \frac{(1 - \alpha - \beta) \gamma_C}{L_C}
\]

(45)

We observe that, as a result of sectoral mobility, immigration into a given sector not only has a negative wage effect and a positive price effect in that sector, but also a negative wage effect and a positive price effect in the other sector. Compared to the case of fixed sector choice, workers experience an additional positive price effect on goods produced in the other sector, represented by the second term in (40) and the first term in (44), by own-sector immigration. They also experience a negative wage effect and a
positive price effect in their own sector, represented by the second term in (41) and the first term in (43), by immigration into the other sector. The old experience an additional positive price effect, represented by the second term in (42) and the first term in (45).

The size of the price and wage effects in sectors A and B is smaller compared to the case of fixed sector choice, because the change in sectoral labor supply due to immigration is smaller: with fixed sector choice, the marginal immigrant increases sectoral labor supply by 1, \( \frac{\partial L_A}{\partial M_A} = 1 \) and \( \frac{\partial L_B}{\partial M_B} = 1 \), while with endogenous sector choice, he increases sectoral labor supply by less than 1, \( 0 < \frac{\partial L_A}{\partial M_A} < 1 \) and \( 0 < \frac{\partial L_B}{\partial M_B} < 1 \), because of the crowding-out effect on native sectoral labor supply. In other words, sectoral mobility mitigates price and wage effects of immigration.

For the voting equilibrium on immigration into the two sectors, we again compute the optimal pairs of immigration levels \((M_A, M_B)\) for each of the three groups of voters. To do so, we solve simultaneously for the respective first-order conditions, i.e. for

- young natives in sector A: \( (40) = 0 \) and \( (43) = 0 \)
- young natives in sector B: \( (41) = 0 \) and \( (44) = 0 \)
- old natives: \( (42) = 0 \) and \( (45) = 0 \),

subject to the lower and upper bound constraints \( M_A > 0, M_B > 0, M_A + M_B < M \).

We know that for the young, the lower constraints on immigration will always be binding, because for any \( M_A \geq 0 \) and \( M_B \geq 0 \), marginal utility from immigration into workers’ own sector is always smaller than marginal utility from immigration into the other sector. Therefore, young natives in sector A choose \( M_B \) by setting \( M_A = 0 \) and solving for \((43)=0\). Young natives in sector B choose \( M_A \) by setting \( M_B = 0 \) and solving for \((41)=0\).

In sum, young natives in sector A choose \((0,M_B^A)\), young natives in sector B choose \((M_B^A,0)\) and old natives choose \((M_A^o,M_B^o)\), where \( M_B^A, M_A^B, M_A^o \) and \( M_B^o \) are equal to the solutions to the first-order conditions in \((43), (41), (42) \) and \((45)\), if positive and smaller in sum than the upper bound \( M \), and equal to zero otherwise.\(^{21}\) There will be a voting equilibrium for zero immigration, if this is the preferred level of immigration for at least two of the three groups of voters.

The only case with a unique voting equilibrium for positive immigration is the one where \( M_B^A > M_A^o > M_A^A \) and \( M_B^o > M_B^B > M_B^B \), analogously to the case for fixed sector choice. This is because there will always be a majority against any other pair of immigration quota than the one preferred by the old \((M_A^o, M_B^o)\).\(^{22}\)

\(^{21}\)Note that the workers who switch sector as a response to immigration are indifferent as the marginal effect on their utility is zero (compare section 4.2).

\(^{22}\)Take the case where \( M_A^o > M_B^A > M_A^A \) and \( M_B^o > M_B^B > M_B^B \). There will always be a majority against any given pair of immigration quota (voting cycle). For example, the young in both sectors will vote against \((M_A^o, M_B^o)\), the young in A and the old will vote against \((M_B^A, M_B^B)\) and the young in B and the old will vote against \((M_A^A, M_B^B)\).
At optimal immigration into each sector, the positive domestic price effects equal the negative foreign price effect. As the negative foreign price effect is the same with or without sectoral mobility, the sum of the two positive price effects in case of sectoral mobility must be just the same as the single positive price effect in case of no sectoral mobility - compare the first-order conditions (21) and (23) with (42) and (45). Therefore, optimal immigration for the old is the same whether native workers are mobile or immobile across sectors, and the outcome of a majority vote that is determined by the old is the same.

To provide a sense for the quantity of results, we report numerical solutions for majority voting outcomes for different parameter values as described in Section 5 below. We find that the negative effects on workers’ wages and on price C dominate the positive price effects of immigration for low rates of domestic consumption (\(e\)), spending shares (\(\alpha, \beta\)) and wage shares (\(\gamma_A, \gamma_B\)). In this case, the young in both sectors vote against immigration not only into their own sector, but also into the other sector (\(M^A_B = 0\) and \(M^B_A = 0\)), and the outcome of a majority vote is \(M_A = 0\) and \(M_B = 0\). We also show cases where the majority voting outcome is indeterminate or positive.

4 Social welfare analysis

In the previous section, we determined the outcome of a majority vote of natives on the amount of sector-specific immigration. In the following, we determine the amount of immigration that is socially optimal for natives, i.e. the amount chosen by a benevolent social planner who simultaneously determines immigration into sectors A and B.

In the following, we use as welfare criterion the sum of individual utilities in the standard utilitarian form:

\[
W(M_i) = v^y_A N_A + v^y_B N_B + v^o N_o, i=A,B. \tag{46}
\]

4.1 Immobile workers

Without sectoral mobility, the marginal social welfare effects of immigration into sectors A and B equal:

\[
\frac{\partial W}{\partial M_A} = \frac{\partial v_A}{\partial M_A} N_A + \frac{\partial v_B}{\partial M_A} N_B + \frac{\partial v^o}{\partial M_A} N_o. \tag{47}
\]

and

\[
\frac{\partial W}{\partial M_B} = \frac{\partial v_A}{\partial M_B} N_A + \frac{\partial v_B}{\partial M_B} N_B + \frac{\partial v^o}{\partial M_B} N_o. \tag{48}
\]
Proposition 3. Without sectoral mobility, socially optimal immigration \((M_A^*, M_B^*)\) equals

\[
\begin{align*}
\left( M \left[ -\frac{1}{\varepsilon} N_A + \alpha \gamma_A (N_A + N_B + N_o) \right] + (N_A + N_B + N_o) [\alpha \gamma_A N_B - N_A (\beta \gamma_B - (1 - \alpha - \beta) \gamma_C)], \\
-\frac{1}{\varepsilon} N_B + \beta \gamma_B (N_A + N_B + N_o) + (N_A + N_B + N_o) [\beta \gamma_B N_A - N_B (\alpha \gamma_A - (1 - \alpha - \beta) \gamma_C)]
\right)
\end{align*}
\]

for \(-\frac{1}{\varepsilon} N_A + \alpha \gamma_A (N_A + N_B + N_o) > 0\) and \(-\frac{1}{\varepsilon} N_B + \beta \gamma_B (N_A + N_B + N_o) > 0\). It equals

\[
\begin{align*}
\left( 0, \frac{M \left[ -\frac{1}{\varepsilon} N_A + \alpha \gamma_A (N_A + N_B + N_o) \right] + (N_A + N_B + N_o) [\alpha \gamma_A N_B - N_A (\beta \gamma_B - (1 - \alpha - \beta) \gamma_C)]}{-\frac{1}{\varepsilon} N_B + \beta \gamma_B (N_A + N_B + N_o) + (N_A + N_B + N_o) [\beta \gamma_B N_A - N_B (\alpha \gamma_A - (1 - \alpha - \beta) \gamma_C)]} \right)
\end{align*}
\]

for \(-\frac{1}{\varepsilon} N_A + \alpha \gamma_A (N_A + N_B + N_o) < 0\) and \(-\frac{1}{\varepsilon} N_B + \beta \gamma_B (N_A + N_B + N_o) > 0\). And it equals

\[
\begin{align*}
\left( M \left[ -\frac{1}{\varepsilon} N_A + \alpha \gamma_A (N_A + N_B + N_o) \right] - (1 - \alpha - \beta) \gamma_C N_A (N_A + N_B + N_o)
\right)
\]

for \(-\frac{1}{\varepsilon} N_A + \alpha \gamma_A (N_A + N_B + N_o) > 0\) and \(-\frac{1}{\varepsilon} N_B + \beta \gamma_B (N_A + N_B + N_o) > 0\), given \(M_A^* > 0\) and \(M_B^* > 0\), respectively. Socially optimal immigration into both sectors is zero, otherwise.

Proof. We simultaneously solve for (49)=0 and (50)=0 with respect to \(M_A\) and \(M_B\) subject to their lower and upper bounds \(M_A \geq 0\), \(M_B \geq 0\) and \(M_A + M_B \leq M\). Given \(M_A \geq 0\), \(M_B \geq 0\) and \(M_A + M_B \leq M\), the marginal welfare effect of immigration into sector A can only be positive, if \(-\frac{1}{\varepsilon} N_A + \alpha \gamma_A (N_A + N_B + N_o) > 0\) and \(-\frac{1}{\varepsilon} N_B + \beta \gamma_B (N_A + N_B + N_o) > 0\).
 Analogously, the marginal welfare effect of immigration into sector B can only be positive, if \(-\frac{1}{\epsilon} N_B + \beta \gamma_B (N_A + N_B + N_o) > 0\), as can be seen by transforming (49) and (50).

Simulation results reported in Table 1 show that socially optimal amounts of immigration \(M^*_A\) and \(M^*_B\) are smaller than the respective majority voting outcomes, which correspond to the preferences of the old \(M^o_A\) and \(M^o_B\). This is because the old do not take the negative effect of immigration on wages into account.

### 4.2 Mobile workers

With sectoral mobility, the marginal social welfare effects of immigration into sectors A and B equal:

\[
\frac{\partial W}{\partial M_A} = \frac{\partial v_A}{\partial M_A} N \Phi (w_A - w_B) + v_A N \Phi \left( \frac{\partial w_A}{\partial M_A} - \frac{\partial w_B}{\partial M_A} \right) + \frac{\partial v_o}{\partial M_A} N_o. \tag{51}
\]

And

\[
\frac{\partial W}{\partial M_B} = \frac{\partial v_A}{\partial M_B} N \Phi (w_A - w_B) + v_A N \Phi \left( \frac{\partial w_A}{\partial M_B} - \frac{\partial w_B}{\partial M_B} \right) + \frac{\partial v_o}{\partial M_B} N_o. \tag{52}
\]

Using (40)-(45) to substitute in (51) and (52) gives

\[
\frac{\partial W}{\partial M_A} = \left[ \alpha \gamma_A \frac{\partial L_A}{\partial M_A} + \beta \gamma_B \frac{\partial L_B}{\partial M_A} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C} \right] (N_A + N_B + N_o) - \frac{1}{\epsilon} \left[ \frac{N_A}{L_A} \frac{\partial L_A}{\partial M_A} + \frac{N_B}{L_B} \frac{\partial L_B}{\partial M_A} \right] + (v_A - v_B) N \Phi \left[ - \frac{\psi_A \gamma_A}{L_A^2} \frac{\partial L_A}{\partial M_A} + \frac{\psi_B \gamma_B}{L_B^2} \frac{\partial L_B}{\partial M_A} \right]. \tag{53}
\]

And

\[
\frac{\partial W}{\partial M_B} = \left[ \alpha \gamma_A \frac{\partial L_A}{\partial M_B} + \beta \gamma_B \frac{\partial L_B}{\partial M_B} - \frac{(1 - \alpha - \beta) \gamma_C}{L_C} \right] (N_A + N_B + N_o) - \frac{1}{\epsilon} \left[ \frac{N_A}{L_A} \frac{\partial L_A}{\partial M_B} + \frac{N_B}{L_B} \frac{\partial L_B}{\partial M_B} \right] + (v_A - v_B) N \Phi \left[ - \frac{\psi_A \gamma_A}{L_A^2} \frac{\partial L_A}{\partial M_B} + \frac{\psi_B \gamma_B}{L_B^2} \frac{\partial L_B}{\partial M_B} \right], \tag{54}
\]
where we can further substitute for the change in total sectoral labor supply due to immigration using (33)-(36).

The first square bracket gives the price effects of immigration: positive effects on prices A and B and a negative effect on price C, affecting all three groups of voters: the young in sectors A and B as well as the old. The second term gives the negative wage effects on the young in sectors A and B. The third term gives the effect on utility of the young that switch sector due to immigration, for given wages. We can show that the third square bracket is negative - that is, immigration into sector A reduces wage A more strongly than wage B. As a consequence, natives switch from sector A into sector B. Now, given equal job conditions in both sectors, utility is higher in the sector where the wage is higher. However, we have assumed that wage differentials compensate for job conditions that are worse in the sector where the wage is higher. Therefore, at the margin, utility derived from wages adjusted for job conditions does not change for those who switch sector.

In order to solve for socially optimal immigration in the case of endogenous sector choice, we again simultaneously solve for (53)=0 and (54)=0 with respect to $M_A$ and $M_B$ subject to their lower bounds $M_A \geq 0$, $M_B \geq 0$ and upper bound $M_A + M_B \leq M$. The solutions to $M_A$, $M_B$ are quadratic due to the quadratic terms in (53) and (54). We report numerical solutions in Table 1 below. As before in the case of immobile workers, we find that socially optimal amounts of immigration $M^*_A$ and $M^*_B$ are smaller than the respective majority voting outcomes determined by the old, $M^*_oA$ and $M^*_oB$, who do not take the negative wage effects of immigration into account.

**Corollary 1.** Socially optimal immigration into a given sector is greater when native workers are mobile than when they are immobile, if the price effect net of the wage effect on social welfare is smaller in that sector than in the other sector. It is smaller otherwise.

**Proof.** The socially optimal amount of immigration into a given sector can be greater or smaller with immobile than with mobile native workers. The marginal social welfare effect of immigration into sector A (B) is greater (smaller) with immobile than with mobile workers, if the positive price effect (net of the negative wage effect) in A is greater than the positive price effect (net of the negative wage effect) in B:

$$\frac{\alpha \gamma_A}{L_A} (N_A + N_B + N_o) - \frac{N_A}{e L_A} > \frac{\beta \gamma_B}{L_B} (N_A + N_B + N_o) - \frac{N_B}{e L_B},$$

as can be derived from a comparison of (49) with (53) and (50) with (54). Thus, the crowding-out effect of immigration into a given sector only proves to be beneficial, if there is more to be gained from immigration (or an increase in the labor force, for that matter) in the other sector.

---

23The third term comes from the fact that we compute the derivative of the products of group-specific indirect utilities with group sizes, both of which depend on immigration. Applying the product rule, we get the derivative of indirect utilities times group size (terms 1 and 2) plus the derivatives of group sizes times indirect utilities (term 3).

24$v_i(x^{i+}) = v_i(x^{i-})$, where $x$ is the consumption good bundle purchased with money and expressed in terms of sector-specific wages and the prices, and $z$ is the compensating variation for working in sector $i$ and not in the other sector. Compare Rosen (1986).
To sum up, immigration is socially more desirable in an immobile than in a mobile society, if the crowding-out of the native labor force reduces welfare, and it is less desirable otherwise. It follows that immigration is not necessarily a substitute for native mobility across sectors. Any optimal policy must raise the labor force in that sector where the gains are highest (for example where the existing labor force is smallest). The labor force can be raised via immigration directly or indirectly, if it causes natives to switch sector. Note that in the absence of immigration, an enhancement of sectoral mobility of natives into a given sector (for example via a policy that reduces the cost of moving) can similarly increase welfare, if the price effect (net of the wage effect) is greater in that sector than in another. However, as long as society benefits from a greater labor force (because positive price effects outweigh negative wage effects), immigration will be more efficient in terms of social welfare.

5 Simulation of immigration quota

Optimal immigration quota derived above depend on the parameter values of our stylized model. In the case of endogenous sector choice, the expressions for optimal immigration are complex enough to best be derived numerically. We therefore use existing parameter estimates to provide a sense of the magnitude of optimal immigration into sectors, both for the groups of young and old, as well as for society overall. This way, we can also easily compare the outcome of a majority vote with the social optimum. Since our stylized model is one of two countries, one home and one foreign, we think of immigration quota as quota for one foreign country.

5.1 Parameter choice

For the relative size of the native labor force $N = N_A + N_B$ and the foreign labor force $M$ that comprises potential immigrants, we choose a ratio of 1. Depending on the countries considered, this ratio can of course be greater or smaller than $1^25$, and we will vary the relative size of the native and foreign labor force in the simulations to show its effect on results. We choose $N = 1$ such that results on immigration quota can be easily interpreted as shares of the total native labor force. We further assume that the number of workers in the tradables sector $B$ and in the non-tradables sector $A$ is the same and use $N_A = 0.5$ and $N_B = 0.5$. The relative size of the old within the native population is set to $N_o = 0.9$ since, according to ILO (2007) statistics, the share of the retired is almost equal to that of the working population in a typical OECD country.$^{26}$

For spending shares in consumption, we use estimates of the shares of spending on tradables and non-tradables in a typical OECD country from the Penn World Table. For domestic demand, we use $\alpha = 0.5$ for the non-tradable good produced in sector $A$ and $\beta = 0.4$ for the tradable good produced in sector $B$, $^{25}$ The labor force in Mexico, for example, is smaller than in the U.S. $^{26}$ This implies that a constant share of 10 per cent of the young die before they become old, as we abstract from population growth in our model.
which implies $1 - \alpha - \beta = 0.1$ for the imported good produced in sector C. For foreign demand, we use $\theta = 0.5$ for the good imported from sector B, which implies $1 - \theta = 0.5$ for the good produced in sector C. For wage shares in production, we use estimates for a typical OECD country from ILO (2000). We set $\gamma_A = 0.7$, $\gamma_B = 0.7$ and $\gamma_C = 0.7$ in the three sectors A, B, C.

We choose a discount rate $\delta$ of 1.3 that corresponds to a yearly discount rate of around 0.02 for the length of a working life of 40 years and a consumption rate $e$ of 0.7. For the cost of capital $r$, we use 0.15.\(^{27}\)

We calibrate $p_A$ and, as implied by relative prices, $p_B$ and $p_C$ as well as capital stocks $K_A$, $K_B$ and $K_C$ according to a previous-period interest rate $r_t-1$ of 0.15, equal to the current interest rate.

We also show simulations for different values of the relative size of the native labor force, the labor intensity in domestic production and the consumption rate.

Table 2 summarizes the baseline parameter values described above and reports corresponding equilibrium values of the model.

### 5.2 Simulation results

Table 1 reports simulation results on immigration quota for no sectoral mobility (panel a) and for sectoral mobility (panel b) of native workers. For each case, we show group-specific quota as well as the social optimum for immigration into both sectors A and B. Quota that are preferred by a majority and, therefore, correspond to the majority voting outcome are in bold. For no sectoral mobility, we find that old natives occupy the median position, as stated in Proposition 1. They prefer more immigration into a given sector than young natives working in that sector, but less immigration than young natives working in the other sector. Their preferred amount of immigration is strictly positive for all chosen parameter values, whereas socially optimal immigration is positive only in columns 4 and 5 (see discussion below).

For sectoral mobility, we find that depending on parameter values there is a majority for zero immigration into both sectors (columns 1 and 2), a voting cycle (column 3) or a voting outcome that is dominated by the old (columns 4 and 5), as stated in Proposition 2. We also find that the optimal amounts of immigration for the old are the same as in the case of no sectoral mobility. The young in both sectors still vote against immigration into their own sector, because the positive price effects from immigration are still not large enough to cover the negative wage effect. Besides, they may now also vote against immigration into the other sector, if the negative effect on their wage due to sectoral crowding-out is not compensated by the two positive price effects. In any case, they will vote for a smaller amount of immigration into the other sector compared to the case of no sectoral mobility. This is because the positive price effect in that sector is now smaller, while the (new) positive price effect in their own sector is dominated by the (new) negative wage effect in their own sector.\(^{28}\)

In columns 2-5 of the table, we show results for variations of parameter values, so that they can be compared with results for the baseline in column 1. In column 2, the native work force in sector B is

\(^{27}\)The cost of capital is the sum of the world interest rate and the rate of capital depreciation and the same as the one used in Kohler and Felbermayr (2007).

\(^{28}\)Compare (21) with (41) for immigration into sector A and (23) with (43) for immigration into sector B.
greater than in the baseline: $N_{A} = 0.6$. We can see that, as a result, old natives and young natives in sector B prefer less immigration into sector A in case of no sectoral mobility. The reason is that the marginal effect of immigration into sector A on the price in that sector is now smaller for any given level of immigration. Since the old and the young in sector B choose immigration into sector B such that marginal price effect in A equals the marginal price effect in C, which remains the same, optimal immigration into sector A is smaller. Note also that optimal immigration into sector B for the old is greater, because optimality requires that the marginal price effects in sectors A and B are the same (both are smaller). Overall, optimal levels of immigration for the old are smaller, the greater the size of the native relative to the foreign labor force, to compensate for the decrease in the positive domestic price effects relative to the negative foreign price effect of immigration. With sectoral mobility, optimal immigration for the old is the same in both sectors as without sectoral mobility because due to the crowding-out effect of immigration on natives domestic price effects remain the same. Optimal immigration for the young is zero in both sectors as the negative wage effects dominate the positive price effects. Therefore, the majority voting outcome on immigration in both sectors is zero just as in the baseline in column 1.

In column 3, we choose a greater wage share in sector B, $\gamma_{B} = 0.8$. In this case, the effect of a price decrease in sector B is greater, and optimal immigration into sector B increases for both the old as well as the young in sector A in case of no sectoral mobility. As a consequence, optimal immigration into sector A decreases for the old, who equate marginal price effects in A and B. For sectoral mobility, an increase in $\gamma_{B}$ increases the positive effect on the price in sector B relative to the negative effect on the price in sector C. These effects large enough for natives in sector B to derive positive net marginal utility from immigration into sector A, whereas net marginal utility from immigration into sector B is still negative for natives in sector A. As a consequence, natives in B now vote for positive immigration into sector A (even though their preferred amount is smaller compared to the case with no sectoral mobility), and natives in A vote for zero immigration into sector B as before. The outcome of a majority vote on immigration into the two sector is indeterminate, as there will be a majority against any given pair of immigration levels that is proposed.

Last, we consider an increase in the consumption rate: $e = 0.9$ in column 4. As a result, the negative effect of immigration on wages decreases. Without sectoral mobility, the wage effect of immigration into one’s own sector always dominates the positive price effect as long as $e \leq 1^{29}$, while immigration into a given sector has no wage effect in the other sector or on the old. Therefore, preferred levels of immigration remain the same. With sectoral mobility, the young in each sector prefer a greater amount of immigration into the other sector compared to the baseline. They also prefer a greater amount than the old, who in turn prefer a greater amount than the young in the sector of immigration (whose preferred amount remains zero). In this case, the majority voting outcome is determined by the old, who prefer strictly positive immigration into both sectors. The decrease in the negative wage effects due to a large consumption rate also causes the social net welfare effect of immigration into sector A to become positive. The decrease in wage effects does not suffice for the net welfare effect of immigration into sector B to

\footnote{29Compare Section 3.}
become positive, because the positive price effect in B is smaller than in A due to a smaller domestic spending share \((\alpha > \beta)\). We also find that socially optimal immigration in A is slightly smaller with than without sectoral mobility (compare Corollary 1).

In column 5, we switch parameter value for domestic spending shares such that the spending share in B is now greater than in A: \(\beta > \alpha\), while the consumption rate remains the same as in column 4. As a result, preferred levels of immigration are reversed in case of no sectoral mobility: the level of immigration that was preferred for sector A in column 4 is now preferred for sector B and vice versa. In case of sectoral mobility, the young in each sector still prefer a positive amount of immigration into the other sector that is greater than the amount preferred by the old, but levels are different. This is because with a change in sectoral spending shares, the effect of immigration on the sectoral distribution of the native labor force changes\(^{30}\) and, therefore, price and wage effects change. In consequence, socially optimal amounts of immigration change as well, and optimal immigration into sector B is now positive while optimal immigration into sector A is zero. Further, socially optimal immigration in B is now slightly greater with than without sectoral mobility.

The simulation results show that the relative effect of immigration on domestic wages and prices is important not only for individual welfare of the young but also for social welfare overall. In particular, unless the decline in real wages is quite small (for example due to a large consumption rate), optimal immigration is zero for the young and for the total population. Note that this also results from the fact that we do not consider any second-generation effects of immigration. As immigrants increase the labor force only in one period and retire or return in the next period, they decrease domestic prices only in the first period but decrease income in both the first and (via savings) in the second period.

\[ 6 \quad \text{Conclusion} \]

We determine the outcome of a majority vote on immigration into sectors as well as the socially optimal amounts. We assume that natives require a compensating wage differential for working in one sector rather than in another, mapping situations of sectoral labor supply shortages that are present in many OECD countries. To analyze immigration policy in this context, we focus on immigrants who are selected to serve as substitutes for natives in a given sector and determine the outcome of a majority vote on sectoral immigration as well as optimal immigration quota. We identify both the wage and the price effects of immigration in an overlapping generations model with three different groups of voters: young natives working in sector A, young natives working in sector B and old natives.

We find that the young are against immigration into their own sector, because the negative wage effect always dominates the positive price effect. The old are affected by immigration only via price effects but not via wage effects and, therefore, support immigration. Finally, if natives are not mobile across sectors, the young are affected by immigration into the other sector only via price effects but not via wage effects.

\(^{30}\) Compare (33)-(36).
wage effects just as the old. Therefore, while they are against immigration into their own sector, they support a positive amount of immigration into the other sector together with the old. The old turn out to represent the median voter and determine the outcome of a majority vote on strictly positive immigration into both sectors.

If natives are mobile across sectors, immigration into a given sector does not only have price and wage effects in that sector but also in the other sector. Then, the young in that sector vote for a smaller amount of immigration into the other sector compared to the case with no sectoral mobility. This amount of immigration can be positive or zero, depending on parameter values. As a result, the outcome of a majority vote on sectoral immigration can be zero, indeterminate or strictly positive and determined by the old, in which case the voting outcome is the same as with no sectoral mobility.

Social welfare effects of immigration are greater the smaller the negative wage effects and the larger the positive price effects. Using existing parameter estimates to simulate optimal immigration quota for a range of plausible parameter values, we find that sectoral immigration becomes optimal only for high rates of domestic consumption (which reduce the negative wage effects), together with high spending shares on domestic sectors and high domestic wage shares in production (which increase the positive price effects). Optimal quota can be greater with or without sectoral mobility depending on parameter values, but they are always smaller than the respective majority voting outcome, if that is positive.

The model is based on a number of simplifying assumptions that allow for the derivation of our results. We abstract from population dynamics and assume a constant ratio of the native to foreign labor force. As shown in Section 5, optimal immigration can be expected to increase if this ratio decreases, for example due to population aging in developed countries. A dynamic model could be used to derive equilibrium outcomes on immigration in the presence of demographic change. We also do not account for any second-generation effects of immigration, or possible complementarities of labor across sectors as for example in Ottaviano and Peri (2006). All of these extensions could potentially increase optimal immigration and represent interesting starting points for future research.
Appendix: Sectoral output values

In Section 2, we derived sectoral equilibrium prices \( p_{i,t} \) as functions of the constants \( \psi_{i,t} \) and sector outputs \( X_{i,t}, i=A, B, C \) in period \( t \):

\[
p_{i,t} = \frac{\psi_{i,t}}{X_{i,t}}. \tag{15'}
\]

We can easily see that the values of sectoral outputs are equal to the constants, expressions of which are given by the following:

\[
\psi_{A,t} = \frac{1}{z} \left[ (\gamma_A \psi_{A,t-1} + \gamma_B \psi_{B,t-1}) \alpha (1 - e^{\gamma_C(1 - \theta)}) + \gamma_C \psi_{C,t-1} e^{\gamma_B \theta} \right] \tag{55}
\]

\[
\psi_{B,t} = \frac{1}{z} \left[ (\gamma_A \psi_{A,t-1} + \gamma_B \psi_{B,t-1})(1 - e^{\gamma_C}) + e^{\gamma_C \theta (1 - \alpha)} + \gamma_C \psi_{C,t-1} (1 - e^{\gamma_A \alpha} \theta) \right] \tag{56}
\]

\[
\psi_{C,t} = \frac{1}{z} \left[ (\gamma_A \psi_{A,t-1} + \gamma_B \psi_{B,t-1})(1 - \alpha - \beta) + e^{\gamma_C \theta (1 - \alpha - \beta)} + \gamma_C \psi_{C,t-1} [(1 - \alpha e^{\gamma_A} - \beta e^{\gamma_B})(1 - \theta) + e^{\gamma_B \theta} (1 - \alpha - \beta)] \right], \tag{57}
\]

where

\[
z = \frac{1 + r_{t-1}}{(2 + \delta)\{(1 - e^{\alpha \gamma_A} - e^{\beta \gamma_B})(1 - (1 - \theta) e^{\gamma_C}) - (1 - \alpha - \beta) e^{2 \gamma_B \gamma_C \theta}\}}.
\]

As the constants and, therefore, sectoral output values, only depend on previous-period output values and exogenous parameters, they are exogenous in any given period and determined by initial sectoral output values \( \psi_{A,0}, \psi_{B,0} \) and \( \psi_{C,0} \).
### Tables

**Tabelle 1: Results on immigration quota (as shares of the total native labor force)**

<table>
<thead>
<tr>
<th></th>
<th>(1) Baseline</th>
<th>(2) Greater native labor force in sector A</th>
<th>(3) Greater wage share in sector B</th>
<th>(4) Greater consumption rate</th>
<th>(5) Greater consumption rate and reverse spending shares on sectors A and B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Immigration into sector A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Group-specific optimum</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Workers in sector A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Workers in sector B</td>
<td>0.75</td>
<td>0.73</td>
<td>0.75</td>
<td>0.75</td>
<td>0.7</td>
</tr>
<tr>
<td>Old natives</td>
<td>0.5</td>
<td>0.45</td>
<td>0.44</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>Social optimum</strong></td>
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<td>0</td>
<td>0</td>
<td>0.17</td>
<td>0</td>
</tr>
<tr>
<td><strong>Immigration into sector B</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Group-specific optimum</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Workers in sector A</td>
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<td>0.7</td>
<td>0.73</td>
<td>0.7</td>
<td>0.75</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Old natives</td>
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<td>0.34</td>
<td>0.36</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Social optimum</strong></td>
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<td>0</td>
<td>0</td>
<td>0.04</td>
</tr>
<tr>
<td><strong>Immigration into sector A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Group-specific optimum</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Workers in sector A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Workers in sector B</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.67</td>
<td>0.62</td>
</tr>
<tr>
<td>Old natives</td>
<td>0.5</td>
<td>0.45</td>
<td>0.44</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>Social optimum</strong></td>
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<td>0</td>
<td>0</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td><strong>Immigration into sector B</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Group-specific optimum</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Workers in sector A</td>
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<td>0</td>
<td>0</td>
<td>0.56</td>
<td>0.62</td>
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<tr>
<td>Workers in sector B</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Old natives</td>
<td>0.3</td>
<td>0.34</td>
<td>0.36</td>
<td>0.3</td>
<td>0.5</td>
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<tr>
<td><strong>Social optimum</strong></td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Note: Equilibrium (median voter elected) outcomes are in bold. There is no equilibrium outcome in case of sectoral mobility in column (3), see Proposition 2.

In column (2), the native labor force in A is $N_A = 0.6$. In column (3), the wage share in sector B is $\gamma_B = 0.8$. In columns (4) and (5), the consumption rate is $\epsilon = 0.9$. In column (5) the domestic spending shares on sectors A and B are reversed: $\alpha = 0.4$ and $\beta = 0.5$. 

---

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Tabelle 2: Benchmark - Calibration and Equilibrium

<table>
<thead>
<tr>
<th>Functional forms</th>
<th>Parameters</th>
<th>Quantities</th>
<th>Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Household utility per period: Cobb-Douglas</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Home</td>
<td>$u_I = \alpha \ln x_A + \beta \ln x_B + (1 - \alpha - \beta) \ln x_C$</td>
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<td></td>
</tr>
<tr>
<td>good produced in sector A</td>
<td>$\alpha = 0.5$</td>
<td>$X_A^d = 0.6108$</td>
<td>$p_A = 0.7976$</td>
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<tr>
<td>good produced in sector B</td>
<td>$\beta = 0.4$</td>
<td>$X_B^d = 0.4336$</td>
<td>$p_B = 0.8989$</td>
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<tr>
<td>good produced in sector C</td>
<td>$(1 - \alpha - \beta) = 0.1$</td>
<td>$X_C^d = 0.2884$</td>
<td>$p_C = 0.3378$</td>
</tr>
<tr>
<td><strong>Foreign</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>good produced in sector B</td>
<td>$\theta = 0.5$</td>
<td>$X_B^d = 0.2092$</td>
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<tr>
<td>good produced in sector C</td>
<td>$1 - \theta = 0.5$</td>
<td>$X_C^d = 0.5568$</td>
<td></td>
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<tr>
<td><strong>Technology: Cobb-Douglas</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Home</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>good produced in sector A</td>
<td>$X_A = L_A^{\gamma_A} K_A^{1-\gamma_A}$</td>
<td>$\gamma_A = 0.7$</td>
<td>$X_A^d = 0.6108$</td>
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<tr>
<td>wage share</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>good produced in sector B</td>
<td>$X_B = L_B^{\gamma_B} K_B^{1-\gamma_B}$</td>
<td>$\gamma_B = 0.7$</td>
<td>$X_B^d = 0.6429$</td>
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<tr>
<td>wage share</td>
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</tr>
<tr>
<td>Foreign</td>
<td></td>
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<tr>
<td>good produced in sector C</td>
<td>$X_C = L_C^{\gamma_C} K_C^{1-\gamma_C}$</td>
<td>$\gamma_C = 0.7$</td>
<td>$X_C^d = 0.8453$</td>
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<tr>
<td>wage share</td>
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<tr>
<td><strong>Labor endowment and allocation</strong></td>
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<td></td>
</tr>
<tr>
<td>Home</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>workers in sector A</td>
<td>$N_A = 0.5$</td>
<td>$w_A = 0.6821$</td>
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</tr>
<tr>
<td>workers in sector B</td>
<td>$N_B = 0.5$</td>
<td>$w_B = 0.8090$</td>
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<tr>
<td>retired</td>
<td>$N_o = 0.9$</td>
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<tr>
<td>workers in sector C</td>
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<td>$w_C = 0.1998$</td>
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<tr>
<td><strong>Capital endowment and allocation</strong></td>
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<td></td>
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<tr>
<td>Home</td>
<td></td>
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<tr>
<td>Consumption rate</td>
<td>$\epsilon = 0.7$ ($\delta = 1.3$)</td>
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<tr>
<td>sector A</td>
<td>$p_A X_A'(K_A) = r$</td>
<td>$K_A = 0.9744$</td>
<td>$r = 0.15$</td>
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<tr>
<td>sector B</td>
<td>$p_B X_B'(K_B) = r$</td>
<td>$K_B = 1.1558$</td>
<td>$r = 0.15$</td>
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<tr>
<td>Foreign</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sector C</td>
<td>$p_C X_C'(K_C) = r$</td>
<td>$K_C = 0.5711$</td>
<td>$r = 0.15$</td>
</tr>
</tbody>
</table>

Note: We assume that the native sectoral population and the interest rate remain unchanged from the previous period, and that $p_{A_t-1} = 1$, $p_{B_t-1} = 1.5$, $p_{C_t-1} = 0.8$. Then, $\psi_{A_t-1} = 0.6729$, $\psi_{B_t-1} = 1.2010$ and $\psi_{C_t-1} = 1.9785$. Implied values of sectoral output in period $t$ are $\psi_{A_t} = 0.4872$, $\psi_{B_t} = 0.5779$, $\psi_{C_t} = 0.2855$. 

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References


Institute for Employment Studies, 2006, Employers’ Use of Migrant Labour, Home Office Online Report 03/06.


